Supplementary Appendix

	Treated (T)	Not treated (nT)	Totals
Outcome (+ve)	**a**	**b**	**a+b**
Outcome (-ve)	**c**	**d**	**d+d**
Totals	**a+c**	**b+d**	**a+b+c+d**

The conversion between RR and LR

$$RR_{+ve} = \frac{a/_{a+c}}{b/_{b+d}} = \frac{\Pr(+ve|T)}{\Pr(+ve|nT)}$$

applying Bayes' rule:

$$RR_{+ve} = \frac{\Pr(+ve|T)}{\Pr(+ve|nT)} = \frac{\frac{\Pr(T|+ve)\Pr(+ve)}{\Pr(T)}}{\frac{\Pr(nT|+ve)\Pr(+ve)}{\Pr(nT)}} = \frac{\Pr(T|+ve)}{\Pr(nT|+ve)}\frac{\Pr(nT)}{\Pr(T)} = \frac{\Pr(T|+ve)}{1-\Pr(T|+ve)}\frac{1-\Pr(T)}{\Pr(T)}$$
$$= \frac{\Pr(T|+ve)}{1-\Pr(T|+ve)} \Big/ \frac{\Pr(T)}{(1-\Pr(T))}$$
$$= posttest\ odds_{+ve}\ of\ exposure \big/ pretest\ odds\ of\ exposure = LR_{+ve}$$

Similarly, for outcome or test *-ve*:

$$RR_{-ve} = \frac{c/_{a+c}}{d/_{b+d}} = \frac{\Pr(-ve|T)}{\Pr(-ve|nT)}$$

Again, applying Bayes' rule:

$$RR_{-ve} = \frac{\Pr(-ve|T)}{\Pr(-ve|nT)} = \frac{\frac{\Pr(T|-ve)\Pr(-ve)}{\Pr(T)}}{\frac{\Pr(nT|-ve)\Pr(-ve)}{\Pr(nT)}} = \frac{\Pr(T|-ve)}{\Pr(nT|-ve)}\frac{\Pr(nT)}{\Pr(T)} = \frac{\Pr(T|-ve)}{1-\Pr(T|-ve)}\frac{1-\Pr(T)}{\Pr(T)}$$
$$= \frac{\Pr(T|-ve)}{1-\Pr(T|-ve)} \Big/ \frac{\Pr(T)}{(1-\Pr(T))}$$
$$= posttest\ odds_{-ve}\ of\ exposure \big/ pretest\ odds\ of\ exposure = LR_{-ve}$$

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Further we have that:

$$\frac{RR_{+ve}}{RR_{-ve}} = \left. \frac{\frac{\Pr(T|+ve)}{1-\Pr(T|+ve)}}{\frac{\Pr(T)}{(1-\Pr(T))}} \middle/ \frac{\frac{\Pr(T|-ve)}{1-\Pr(T|-ve)}}{\frac{\Pr(T)}{(1-\Pr(T))}} \right. = \frac{\frac{\Pr(T|+ve)}{1-\Pr(T|+ve)}}{\frac{\Pr(T|-ve)}{1-\Pr(T|-ve)}} = OR_{+ve}$$

Because:

$$OR = \left. \frac{a/_{a+c}}{b/_{b+d}} \middle/ \frac{c/_{a+c}}{d/_{b+d}} \right. = \frac{\frac{\Pr(+ve|T)}{\Pr(+ve|nT)}}{\frac{\Pr(-ve|T)}{\Pr(-ve|nT)}} = \frac{RR_{+ve}}{RR_{-ve}} = \frac{LR_{+ve}}{LR_{-ve}}$$

If we perceive *RR* as a ratio of risks, the risk difference *RD* is:

$$RD = r_1 - r_0 = RR_{+ve} \times r_0 - r_0$$

Where r_0 is the baseline risk $\Pr(+ve|nT) = {}^{b}/_{b+d}$ and r_1 is the risk in the exposed $\Pr(+ve|T) = {}^{a}/_{a+c}$. Indeed:

$$\frac{\Pr(+ve|T)}{\Pr(+ve|nT)}\Pr(+ve|nT) - \Pr(+ve|nT) = RD$$

If we perceive *RR* as the ratio of posterior and prior odds of exposure, we have:

$$OR = \frac{LR_{+ve}}{LR_{-ve}}$$

$$\frac{r_1/_{r_0}}{1-r_1/_{1-r_0}} = \frac{LR_{+ve}}{LR_{-ve}}$$

$$\frac{r_1}{1-r_1} = \frac{r_0}{1-r_0}\frac{LR_{+ve}}{LR_{-ve}}$$

Setting $A = \frac{r_0}{1-r_0}\frac{LR_{+ve}}{LR_{-ve}}$ we have that:

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$$r_1 = {}^{A}\!/_{1+A} = \frac{r_0}{1-r_0}\frac{LR_{+ve}}{LR_{-ve}} \Big/ 1 + \frac{r_0}{1-r_0}\frac{LR_{+ve}}{LR_{-ve}}$$

The latter relationship comes from the fact that for any odds we have that if $odds = \frac{\Pr}{1-\Pr}$ then $\Pr = \frac{odds}{1+odds}$.

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