Supplementary Appendix

<table>
<thead>
<tr>
<th></th>
<th>Treated (T)</th>
<th>Not treated (nT)</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outcome (+ve)</td>
<td>a</td>
<td>b</td>
<td>a+b</td>
</tr>
<tr>
<td>Outcome (-ve)</td>
<td>c</td>
<td>d</td>
<td>d+d</td>
</tr>
<tr>
<td>Totals</td>
<td>a+c</td>
<td>b+d</td>
<td>a+b+c+d</td>
</tr>
</tbody>
</table>

The conversion between RR and LR

\[
RR_{+ve} = \frac{a}{a + c} = \frac{\Pr(T|+ve)}{\Pr(nT|+ve)}
\]

Applying Bayes’ rule:

\[
RR_{+ve} = \frac{\Pr(T|+ve)}{\Pr(nT|+ve)} = \frac{\Pr(T|+ve) \Pr(+ve)}{\Pr(nT|+ve) \Pr(+ve)} = \frac{\Pr(T|+ve)}{\Pr(nT|+ve)} \frac{\Pr(+ve)}{\Pr(+ve)} = \frac{\Pr(T|+ve)}{1 - \Pr(T|+ve)} \frac{1}{\Pr(T)}
\]

\[
= posttest odds_{+ve} of exposure/ pretest odds of exposure = LR_{+ve}
\]

Similarly, for outcome or test -ve:

\[
RR_{-ve} = \frac{b}{b + d} = \frac{\Pr(T|-ve)}{\Pr(nT|-ve)}
\]

Again, applying Bayes’ rule:

\[
RR_{-ve} = \frac{\Pr(T|-ve)}{\Pr(nT|-ve)} = \frac{\Pr(T|-ve) \Pr(-ve)}{\Pr(nT|-ve) \Pr(-ve)} = \frac{\Pr(T|-ve)}{\Pr(nT|-ve)} \frac{\Pr(-ve)}{\Pr(-ve)} = \frac{\Pr(T|-ve)}{1 - \Pr(T|-ve)} \frac{1}{\Pr(T)}
\]

\[
= posttest odds_{-ve} of exposure/ pretest odds of exposure = LR_{-ve}
\]
Further we have that:

\[
RR_{+ve} = \frac{\frac{\Pr(T \mid +ve)}{1 - \Pr(T \mid +ve)}}{\frac{\Pr(T)}{1 - \Pr(T)}} = \frac{\Pr(T \mid +ve)}{1 - \Pr(T \mid +ve)} = OR_{+ve}
\]

Because:

\[
OR = \frac{a/d + c}{b/d + d} = \frac{\frac{\Pr(+ve \mid T)}{\Pr(+ve \mid T)}}{\frac{\Pr(-ve \mid T)}{\Pr(-ve \mid T)}} = \frac{RR_{+ve}}{RR_{-ve}} = \frac{LR_{+ve}}{LR_{-ve}}
\]

If we perceive \( RR \) as a ratio of risks, the risk difference \( RD \) is:

\[RD = r_1 - r_0 = RR_{+ve} \times r_0 - r_0\]

Where \( r_0 \) is the baseline risk \( \Pr(+ve \mid \neg T) = b/\text{b} + d \) and \( r_1 \) is the risk in the exposed \( \Pr(+ve \mid T) = a/\text{a} + c \).

Indeed:

\[
\frac{\Pr(+ve \mid T)}{\Pr(+ve \mid \neg T)} \Pr(+ve \mid T) - \Pr(+ve \mid \neg T) = RD
\]

If we perceive \( RR \) as the ratio of posterior and prior odds of exposure, we have:

\[
OR = \frac{LR_{+ve}}{LR_{-ve}}
\]

\[
\frac{r_1}{r_0} = \frac{LR_{+ve}}{LR_{-ve}}
\]

\[
\frac{r_1}{1 - r_1} = \frac{r_0}{1 - r_0} \frac{LR_{+ve}}{LR_{-ve}}
\]

Setting \( A = \frac{r_0}{1 - r_0} \frac{LR_{+ve}}{LR_{-ve}} \) we have that:
\[ r_1 = \frac{A}{1 + A} = \frac{r_0 \frac{LR_{\text{neg}}}{LR_{\text{pos}}}}{1 - r_0 \frac{LR_{\text{neg}}}{LR_{\text{pos}}}} \]

The latter relationship comes from the fact that for any odds we have that if \( \text{odds} = \frac{\text{Pr}}{1 - \text{Pr}} \) then \( \text{Pr} = \frac{\text{odds}}{1 + \text{odds}} \).